Mathematical description of concrete laying robots

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Abstract –

The object of the research is robots with a manipulation system for concrete-laying operations. The kinematic scheme of a robot with an articulated distributive arm, providing delivery of concrete to any point of the erected object and allowing to bypass various kinds of encountered obstacles, is presented. The closed loops in the form of three- and four-links, which make up the manipulation system, are considered and mathematically described, which is conditioned by the use of a hydraulic drive. On the basis of geometrical approach connections between them are defined, as well as dependences of their velocities and accelerations are established. The description of the dynamics of the manipulating system is made on the basis of the Lagrange method, formulated through the D'Alamber principle, which allowed to obtain the resulting equations in a convenient vector-matrix form. The problem of planning the trajectories of a robotized concrete-laying arm nozzle on the examples of monolithic buildings and structures erection by means of sliding, repositionable and volumetric formwork has been solved. Approximation methods are used to form the time laws of changes in the generalized coordinates of the manipulation system.

Keywords –

Robot; Manipulator; Concrete; Path Planning; Mathematical Modell; Approximation

1 Introduction

Modern construction is characterized by large volumes of concrete work. Monolithic concrete and reinforced concrete are widely used for the construction of chimneys, cooling towers, silos, heavy columns, various tanks, energy facilities, retaining walls, complex arched and vaulted coverings. Monolithic structures are used for construction of high-rise public buildings and residential buildings. Monolithic reinforced concrete is used for the construction of high-rise buildings with complex, expressive plans and combinations of volumes. The development trends of concrete work technology provide a priority solution to the problems of comprehensive mechanization of supply, distribution and placement of concrete mix. At construction sites, machines and equipment are required that would perform continuous feeding and placement of concrete mixture in the structure and ensure a specified rate of concreting. Concrete pumps, concrete placing equipment, concrete placing booms and robots are capable of reducing the laboriousness of concrete placing and transporting the concrete mix, eliminating drastic manual labor and increasing the level of productivity. They are especially effective when concreting large areas and construction of monolithic reinforced concrete buildings.

Analysis of features of the technology of concrete work shows that for distribution and laying of concrete mixture it's advisable to use articulated distribution arms (Fig. 1) having 4-5 degrees-of-freedom. Such structure ensures delivery of concrete to any point of the object being erected and allows you to bypass various kinds of

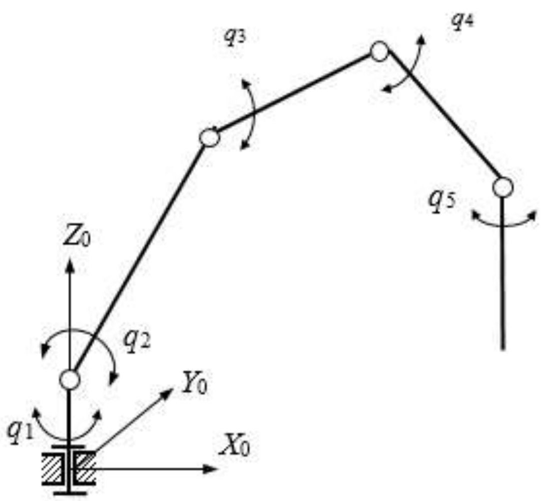

Figure 1. Kinematic diagram of a concrete-laying manipulator

obstacles.

It is known that manipulation systems near special positions (singularities) have sharply reduced load capacity associated with loss of controllability [1,2]. A number of studies are devoted to the consideration of trajectories that do not contain special positions, as well as the definition of the singularity criterion [3,4]. It is worth mentioning that one of the most applicable approaches to solving this issue is based on Jacobi matrices, which express the correlation between the force and kinematic parameters corresponding to a finite link and the input influences - generalized coordinates and forces. In this article, we will not focus on this question, treating it as solved.

For concrete-laying operations performed in monolithic construction, it is reasonable to use a manipulator system located on a rotary tower (Fig. 2). The arm manipulator links are equipped with an electro-

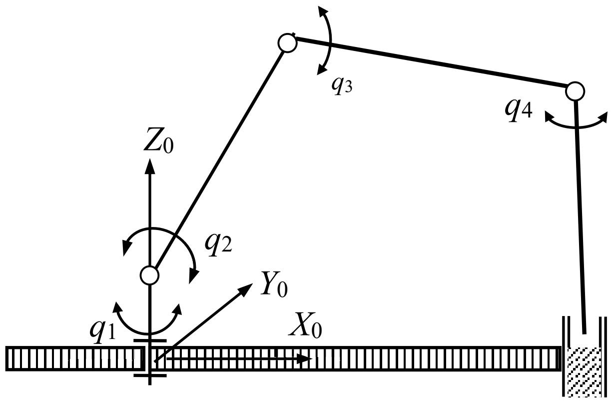

Figure 2. Structure of the articulated arm for concrete laying in the formwork

hydraulic drive. The last link is equipped with a flexible nozzle that produces a horizontal linear movement in any direction. The handling system can cover a wide area and ensures delivery of concrete to any point on the work site.

When laying and distributing concrete over large horizontal areas, it is advisable to use manipulators whose structure is a horizontal multi-link (4-5 links) structure equipped with concrete feeders with a hydraulic drive and remote control (Fig. 3).

The control of concrete-laying manipulators is based on the peculiarities of the objects being erected and the volume of concrete work. For concrete-laying manipulators designed to work in line-of-sight conditions, interactive control from a console located on the manipulator's stand is used. The transition to remote control from the site of work makes it possible to significantly simplify the control of concrete mixer trucks with articulated arms and to increase the quality of concrete mix distribution in the formwork. In this case, radio and infrared control devices are used. The proportional radio remote control system makes it possible to regulate all the functions of the concrete pump and the concrete placing arm very accurately from a distance. This allows the operator to move freely around the construction site and constantly monitor the concrete delivery point. When using radio control equipment at the same time provides the ability to remotely control the equipment via cable. When using radio remote control and radio control, it is necessary to strive for minimizing the control operations of the concrete placing arm, contributing to facilitate the work of the operator. This can be achieved by implementing remote control on a strategic level and by using special controls. For example, joysticks with velocity vector control are used to control the horizontal movement of the robot arm. However, in most cases, preference should be given to software control systems for controlling the movement of the arm-manipulator links. Installation of the link position sensors and ultrasonic sensors for obstacle control in a manipulator makes it possible to ensure program control of the concreting process with the help of microprocessor technology. In this case, it must be provided for remote termination of the manipulator in the program mode and the transition to the command control from the remote control. Taking into account the conditions of movement of manipulator links when laying concrete into the formwork, the presence of reinforcement, jacking rods and equipment in the laying area, concrete-laying robots with adaptive control should be used. To implement adaptive control, it is necessary to solve a number of problems: monitoring the state of the service area, building models of the working environment, adjusting the state of the model in real time, building adaptive control algorithms.

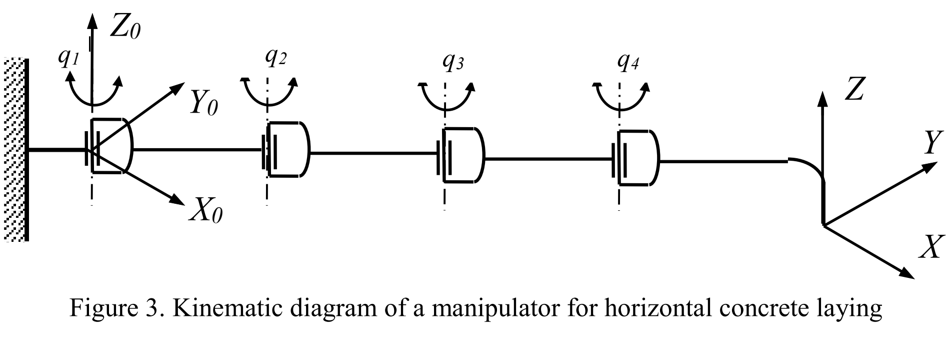

Figure 3. Kinematic diagram of a manipulator for horizontal concrete laying

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The automation of concrete laying requires the automation of the process of compaction of the concrete mixture in the formwork. When using manipulators for concrete laying, the mixture can be compacted by remotely controlled vibrators installed at the output end of the concrete conveyor. High economic performance gives the use of arm manipulators, equipped with compaction devices in the construction of high-rise monolithic buildings and structures in the repositionable and sliding formwork. In this case, the structural arrangement of concrete laying complexes additionally includes concrete compaction and control facilities. Concrete compacting vibrators are installed on the manipulator. It can also be outfitted with tools for finishing work. The installation of link position sensors and ultrasonic sensors for obstacle control in a manipulator makes it possible to control the concreting process by means of microprocessor technology [5,6].

The robotization of concrete-laying operations, based on the use of arm manipulators, should be solved by building robotized concrete-laying complexes (RCLC). Such RCLCs, apart from a concrete-laying manipulator, include concrete conveyors, concrete pumps, receiving hoppers, mechanisms for concrete compaction, robots for laying and welding of reinforcement grids and frameworks.

Many scientific researches and practical developments are devoted to robotization of concrete-laying works using manipulators. At the same time, it should be noted that there is still no systematic approach in solving these problems. It consists primarily in the necessity of mathematical identification of concrete-laying robots and unambiguous representation of them as objects of automatic control. On this basis, it is possible to synthesize various control systems and plan optimal trajectories for the movement of actuators. This article is devoted to the consideration of these issues.

2 Mathematical models of concrete-laying robots

When a hydraulic drive is used in concrete-laying robots, closed loops in the form of three- and four-links are formed (Fig. 4). These mechanisms are described by nonlinear functions and it is necessary to build kinematic and dynamic models to control them. The task of the kinematic analysis of these structures is to establish relations between movements of the drives and the angle of turn of the link, as well as dependences of velocities and accelerations of these parameters. In accordance with the kinematic scheme of the manipulator, local coordinate systems are constructed, on the basis of which the transition matrices from one coordinate system to another $T_{i-1,1}$ are written. These matrices allow to obtain the equations of interrelation of the basic coordinates with the generalized:

$$\bar{x} = f_1(\bar{q}),\ \bar{r}_p = T_{01}T_{12}T_{23}\cdots T_{n-1,n}\bar{r}_p = T_{0n}\bar{r}_p^*$$

where $\bar{r}_p^*$ - position vector of point P in the grip coordinate system; $T_{i-1,1}$ - transformation matrix of coordinate systems. To solve the inverse kinematics problem ($q = f^{-1}(x)$) it is convenient to use recurrence relations of the form:

$$q_{k-1} = q_k + J_n^{-1}(q_k)\cdot(x - f(q_k)), \qquad (1)$$

where $J_n = \partial f / \partial q$ is Jacobi matrix.

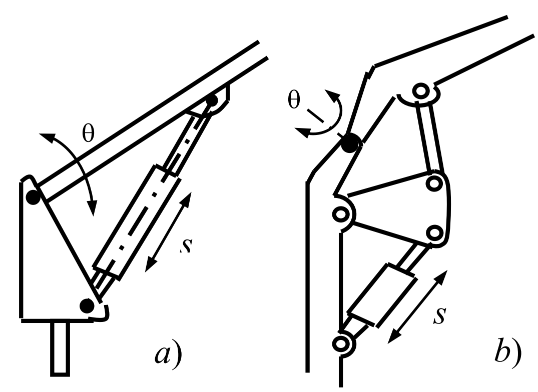

Figure 4. Schematics degrees of mobility of hydraulic arm manipulators

Kinematic models of the considered structures besides the given equations must contain equations of connection of generalized coordinates q_i with displacements of hydraulic actuators s_i: $q_i=f_1(s_i)$, which allow to set dependences between vectors $\bar{x}$ and $\bar{s} : \bar{x} = f_2(\bar{s})$. For the considered structures the required functions $q_i=f(s_i)$ are easily obtained on the basis of the geometric approach. For kinematic structure of the second link with a hydraulic jack, providing lifting and lowering of a jib (fig. 4) this interrelation is described by the equation like:

$$\theta(s) = \mathrm{arctg}(a/b) + \arccos\left(\left(l_1^2 + l_2^2 - s^2\right)/(2l_1l_2)\right) + \mathrm{arctg}(e/d) - \pi, \qquad (2)$$

where a, b, c, d, e – structural parameters; $l_1=\sqrt{a^2+b^2}$ and $l_2=\sqrt{d^2+e^2}$ – interaxial distances. This type of structure, in the first place, refers to the second degree of mobility, where, for the convenience of reporting the generalized coordinate q_2, it is convenient to use an angle γ, equal to $\gamma = \theta + \pi/2 - \mathrm{arctg}(a/d)$. In this case the generalized coordinate is related to the drive displacement by the relation:

$$\theta(s) = \mathrm{arctg}(a/b) + \arccos\left(\left(l_1^2 + l_2^2 - s^2\right)/(2l_1l_2)\right) - \pi/2. \qquad (3)$$

To solve inverse problems about the position of the considered structure, its kinematic model includes coupling equations $s_i = f_2(q_i)$, which are also defined on the basis of the geometric approach:

$$s(\theta) = \left(l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta - \alpha - \varphi)\right). \quad (4)$$

Similarly, the relationship equations between the generalized coordinates *qi* and the hydraulic actuator movements s_i are found: $q_i = f_2(s_i)$ for the kinematic structures representing a Witt chain with the three- and four-links connected in series (Fig. 5b). These relations are obtained by pre-recording the expressions:

$$\varphi_{1i}(s) = \angle O_{1i} O_{2i} O_{3i} = \arccos\left((a_{0i}^2 + a_{1i}^2 - s_i^2)/2a_{0i}a_{1i}\right)$$

$$\varphi_{2i}(s) = \angle O_i O_{2i} O_{4i} = \pi + \alpha_0 + \alpha_1 - \varphi_{1i}(s),$$

$$\varphi_{2i}(s) = \angle O_{2i} O_i O_{5i} = \arccos\left(\frac{d_i^2 + b_{0i}^2 - b_{1i}^2}{2b_{0i}d_i}\right) + \arccos\left(\frac{d_i^2 + b_{3i}^2 - b_{2i}^2}{2b_{3i}d_i}\right),$$

where $d_i(s) = \sqrt{b_0^2 + b_1^2 - 2b_0 b_1 \cos(\varphi_1(s))}$.

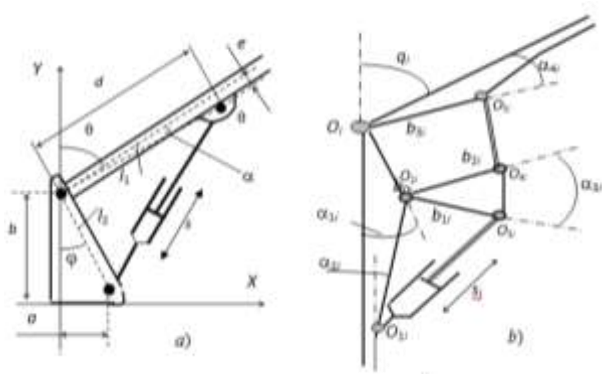

Figure 5. Calculation diagrams of degrees of mobility of a hydraulically driven concrete laying manipulator

Using these expressions, the coupling equations for generalized coordinates are written:

$$q_i(s) = \alpha_{1i} + \alpha_{4i} - \pi + \varphi_{3i}(s). \quad (5)$$

In accordance with the above relations, the dependence has a non-linear nature. However, modeling of the above equations has shown that at certain ratios of design parameters it is possible to obtain dependences close to linear in the operating range of the hydraulic cylinder. In this connection at designing of manipulating mechanisms with the hydraulic drive it is necessary to carry out optimization of a parity of parameters of kinematic structures providing reception of quasi-linear characteristics: $q_i = f_1(s_i)$; $s_i = f_2(q_i)$. It gives an opportunity to carry out linearization of the given equations and to receive convenient relations for construction of control algorithms.

The equations for velocities and accelerations are required to obtain complete kinematic models of the considered structures [6]. The structures under consideration use a linear motion drive, which is converted into angular displacements of manipulator links through multilink elements. The angular and linear displacement velocities of the i-th link are expressed through the velocities of the i-1st link in the form:

$$\bar{\omega}_i = \bar{\omega}_{i-1} + (1 - \xi_i)\bar{e}_i q_i$$
$$\bar{v}_i = \bar{v}_{i-1} + \bar{\omega}_{i-1} \times \bar{r}_{i-1,i} + \bar{e}_i \cdot q'_i \quad (6)$$

Where $\bar{r}_{i-1,i}$ is the vector connecting the axes of degrees of mobility *i-1* and *i-th*; ξ_i is the logical coefficient describing the type of kinematic pair: rotational $\xi=0$; translational $\xi=1$. Linear and angular accelerations of two coordinate systems on the basis of the Coriolis theorem are written in the form

$$\bar{\varepsilon}_i = \bar{\varepsilon}_{i-1} + (1 - \xi_i) \cdot \bar{\omega}_i \times \bar{e}_i \cdot q'_i + (1 - \xi_i) \cdot \bar{e}_i \cdot \bar{q},$$

$$\bar{a}_i = \bar{a}_{i-1} + \bar{\varepsilon}_{i-1} \times \bar{r}_{i-1,i} + \bar{\omega}_{i-1} \times \left(\bar{\omega}_{i-1} \times \bar{r}_{i-1,i}\right) +$$
$$+ \xi_i \cdot \bar{e}_i \cdot q''_i + 2\xi_i \cdot \bar{\omega}_i \times \bar{e}_i \cdot q',$$

where $\bar{\varepsilon}_{i-1}$, $\vec{\varepsilon}_i$ are vectors of angular accelerations of *i-1* and *i-th* manipulator link; $\bar{a}_{i-1}$, $\bar{a}_i$ are vectors of linear accelerations of *i-1* and *i-th* manipulator link. Final dependences of linear and angular velocities as functions of generalized velocities and generalized coordinates are obtained as sums of:

$$\bar{\omega}_i = \sum_{j=1}^{n} \bar{\omega}_i^j q'_j, \quad \bar{v}_i = \sum_{j=1}^{n} \bar{v}_i^j q'_j, \quad \bar{\omega}_i^j = \sum_{j=1}^{n} (1 - \xi_i)\bar{e}_j,$$

$$\bar{v}_i^j = \bar{\omega}_i^j \times \bar{r}_{j,i} + \xi_j \bar{e}_j. \quad (7)$$

Similarly, we can write down the final expressions for accelerations:

$$\bar{e}_i = \sum_{j=1}^{n} (\bar{\omega}_i^j q''_j + (\sum_{k=1}^{n} (\bar{\omega}_i^k \times \bar{\omega}_i^j) q'_k) q'_j). \quad (8)$$

The description of drive mechanisms in kinematic models is represented by a system of equations linking drive state parameters with generalized coordinates, velocities and accelerations. The generalized coordinates are calculated from the drive coordinates based on the following equations:

$$\partial q_i = k_{ji}^{\dot{s}} \partial s_i, \quad k_{ji}^{\dot{s}} = \frac{\partial f_s(s_i)}{\partial s_i}. \quad (9)$$

The description of the generalized velocities and accelerations of the robot is conveniently represented in the form of a vector-matrix equation:

$$\bar{q}' = k_{\dot{q}}^{\dot{s}} \cdot \bar{s}' = \mathrm{diag}\left\{\frac{\partial f_s(s)}{\partial s}\right\}. \quad (10)$$

By differentiating the last equations, we obtain the coupling equation for accelerations:

$$\bar{q}'' = k_{\dot{q}}^{\dot{s}} \bar{s}'' + (k_{\dot{q}}^{\dot{s}})' \bar{s}' = k_{\dot{q}}^{\dot{s}} \bar{s}'' + \mathrm{diag}\left\{\frac{\partial^2 f_s(s)}{\partial s^2}\right\} \cdot (\bar{s}')^2 \quad (11)$$

For the structural scheme of type 1 (Fig. 4a), the equations of generalized and drive velocity relations are represented in the form:

$$\theta'(s') = k_{\dot\theta}^{(1)}(s) \cdot s' = \frac{s \cdot s'}{(4l_1^2 l_2^2 - (l_1^2 + l_2^2 - s^2)^2)^{1/2}}, \quad (12)$$

$$s'(\theta') = k_{\dot s}^{(1)}(\theta) \cdot \theta' \frac{l_1 l_2 \sin(\alpha + \varphi - \theta) \cdot \theta'}{(l_1^2 + l_2^2 + 2 l_1 l_2 \cos(\theta - \alpha - \varphi))^{1/2}}.$$

Similarly, the equations of generalized and drive-velocity relations for the structural scheme of type 2 can be written down (Fig. 4b):

$$\theta'(s') = k_{\dot\theta}^{(1)}(s)s', \; s'(\theta') = k_{\dot s}^{(1)}(\theta)\,\theta'.$$

The structural features of concrete-laying robots complicate the construction of their dynamic models. The analysis of methods for describing the dynamics of manipulating mechanisms has shown that, for concrete-laying robots based on articulated arms, it is expedient to build dynamic models using a method based on the Lagrange method formulated through the D'Alambert principle [7]. This makes it possible to obtain the resulting equations in a convenient vector-matrix form. For the manipulation system of concrete-laying robots with closed kinematic chains of hydraulic actuators, the balance of virtual forces is as follows:

$$\sum_{i=1}^{n}\sum_{v=0}^{N_i}\{m_{iv}(\ddot{\mathbf{r}}_{iv} - \bar{\mathbf{g}})\delta\mathbf{r}_{iv} + (\mathbf{I}_{iv}\varepsilon_{iv} + \omega_{iv} \times \mathbf{I}_{iv}\omega_{iv})\delta\gamma_{iv}\} = \sum_{i=1}^{n}\tau_i \delta q_i, \quad (13)$$

where r_i is vector-vector of link *i* with mass *m* and moment of inertia *j* in the point of gravity; $\bar{\mathbf{g}}$ is vector of gravitational interaction; *τ* is drive moment of drive forces in the *i-th* link; *N* is a number of links of the *i-th* kinematic pair.

For a virtual representation of the movement of manipulator links, the equations of dynamics should be applied in the form of:

$$\sum_{k=1}^{n}\sum_{v=0}^{N_i}\{m_{kv}v_{kv}(\ddot{\mathbf{r}}_{kv} - \bar{\mathbf{g}}) + (\mathbf{I}_{kv}\varepsilon_{kv} + \omega_{kv} \times \mathbf{I}_{kiv}\omega_{iv})\delta\gamma_{kv}\} = \sum_{i=1}^{n}\tau_k \delta q_k. \quad (14)$$

Equations in drive coordinates are needed to control the drive coordinates. To obtain them, use the virtual work balance equations in articulated coordinates and the relationship equations between the generalized coordinates and the drive coordinates. This results in dynamic equations in drive coordinates:

$$\mathbf{M(s)\ddot{s} + h(s,\dot{s}) + g(s) = F}_\mathrm{A}, \quad (15)$$

where $\mathbf{M(s)} = k_{\dot q}^{i}\mathbf{M(q)}k_{\dot q}^{i} + \mathbf{h(s,\dot s)} + \mathbf{g(s)} = \mathbf{F}_\mathrm{A}$;

$\mathbf{h(s,\dot s)} = \mathbf{k}_{\dot q}^{i}\mathbf{M(q)}\mathbf{k}_{\dot q}^{i}\dot s^2 + \mathbf{k}_{\dot q}^{i}\mathbf{h(q,\dot q)} = \mathbf{k}_{\dot q}^{i}\mathbf{M(q)}\dot{\mathbf{k}}_{\dot q}^{i}\dot s + \mathbf{k}_{\dot q}^{i}\mathbf{h(q,\dot q)}$;

$\mathbf{g(s)} = \mathbf{k}_{\dot q}^{i}\mathbf{g(q)}$.

An analysis of the dynamic properties of a concrete-laying manipulator requires the construction of a dynamic model of the complex system, taking into account the dynamics of the hydraulic drives. In addition to modeling of individual components, it is important to describe the relationship between the mechanical and hydraulic parts of the manipulator, because on the one hand the dynamic characteristics of the mechanical subsystem is determined by the resulting forces of the hydraulic drives, and on the other hand the impact of the mechanical part on the hydraulic drives has a significant impact on their own dynamics.

In the process of modeling such a drive, its static and dynamic models are considered, as well as the simulation of the hydraulic cylinder friction is performed. The main functional blocks of the hydraulic part of the actuator are a double-acting power hydraulic cylinder, an electromechanical converter and a two-stage electro-hydraulic amplifier that includes a three-position hydraulic distributor and a jet amplifier of "nozzle-slide" type. Feedback is provided by the hydraulic motor load. The hydraulic directional control valve, which is hydraulically controlled and determines direction of flow and flow rate of the fluid supplied to the hydraulic cylinder, has a significant influence on static and dynamic characteristics of the electro-hydraulic drive.

3 Path planning of concrete-laying robots

Let's consider the planning of concrete-laying robots trajectory on the examples of monolithic buildings and structures erection by means of sliding, repositionable and volumetric formwork. Analysis of the construction of monolithic pipes, silos, building cores and residential buildings showed that laying the next layer of concrete is done in the horizontal plane. Therefore, the problem of constructing a trajectory of the nozzle of the robotic concrete-laying arm is reduced to the description of the sequence of motions in the plane $X_p Y_p$ parallel to the plane $X_0 Y_0$ of the coordinate system of the robot. The coordinate Z_p does not change within one laying cycle and then increases by the value Δh corresponding to the step of the formwork raising. Fig. 6 shows examples of the most frequently erected monolithic objects. As can be seen from the figures the movement trajectories are either a set of rectangles or circles. When erecting the frame of a residential building (Fig. 6a), it is necessary to formulate an array of coordinates of points $P_1, P_2, \ldots P_{10}$ on the basis of the building plan and determine the sequence of rectilinear sections $P_i \rightarrow P_j$: for sections parallel to the axis $X_0 \rightarrow P_{js}(x_{js}, y_{js}), P_{je}(x_{je}, y_{je})$

$$y = y_{js} \vee y_{j\kappa} \rightarrow x_{js} \le x \le x_{je}; \quad (16)$$

and for sections parallel to the axis $Y_0 \rightarrow P_{is}(x_{is}, y_{is}), P_{ie}(x_{ie}, y_{ie})$

$$x = x_{is} \mathrm{V}\, x_{ie} \rightarrow y_{is} \le y \le y_{ie} . \qquad (17)$$

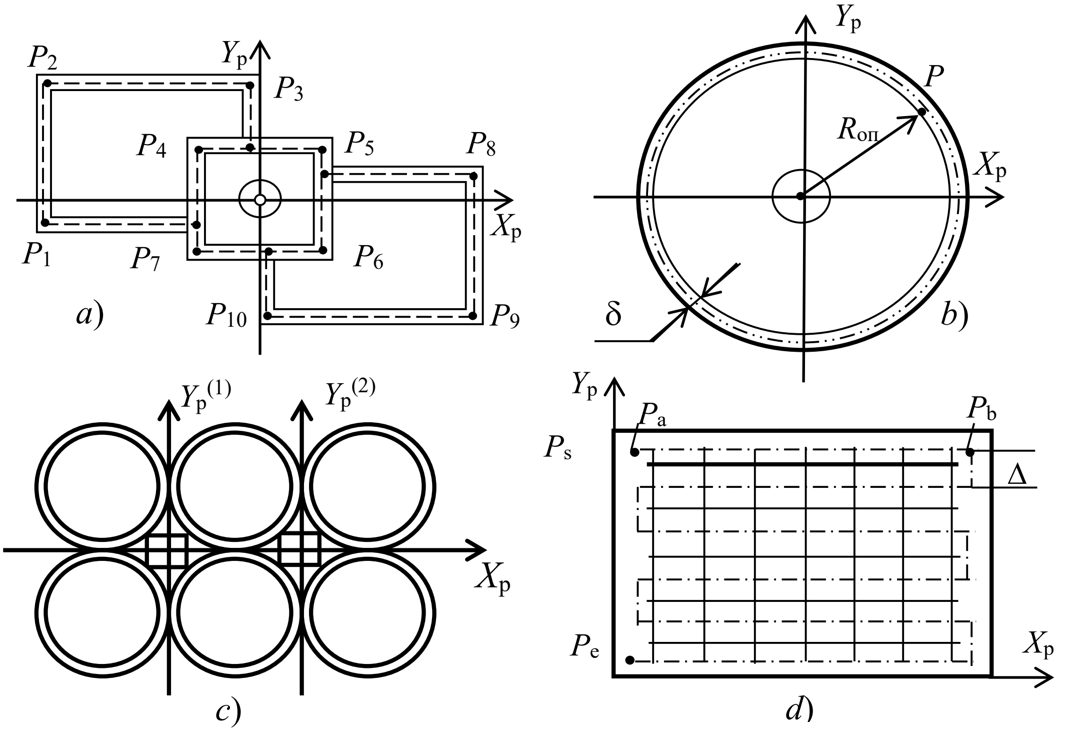

Figure 6. Plans of monolithic objects and concrete laying paths

When erecting chimneys, the concrete-laying robot is positioned on a load shaft in the center of the structure. Therefore, its trajectory is a circle of radius $R_{\text{ср}}$ lying in the horizontal plane $X_{\text{м}}Y_{\text{м}}$ of concrete-laying works is performed on the basis:

$$(x - \Delta x_{\text{см}})^2 + (y - \Delta y_{\text{см}})^2 = R^2{}_{\text{ср}} ,$$

where $\Delta x_{\text{см}}, \Delta y_{\text{см}}$ is the deviation of the formwork from the design axis.

When planning the trajectories of concrete-laying robots, there are a number of cases that require the use of approximation methods to form the time laws of variation of the generalized coordinates:

$$q(t) = [q_1(t),\ q_2(t), ..., q_n(t) \cdot$$

The first of them is associated with displacement of a working body from the initial stationary state, where $\dot{q}_{\text{s}} = \ddot{q}_{\text{s}} = 0$, to a given final state $\dot{q}_{\text{e}} = \ddot{q}_{\text{e}} = 0$.

The problem is to form a law *q(t)* using the given vectors of generalized coordinates at the end points of the trajectory and the zero values of velocities and accelerations of generalized coordinates at these points. When solving this problem, the limit conditions imposed on the derivatives of the generalized coordinates must be taken into account:

$$q_i^{(k)}(t) \le (q_i^{(k)})_{\text{add}} (k = 1,2...m) .$$

The limits $(q_i^{(k)})_{\text{add}}$ are determined by the maximum forces and moments developed by the robot drives. Given that there are six boundary conditions in this case, we can obtain a function $q_j(t)$ for each generalized coordinate in the form of a polynomial of order 5:

$$q_j(t) = \sum_{k=0}^{5} a_{jk} t^k \rightarrow j = 1, 2, ...,\ n \cdot$$

The coefficients of the interpolation polynomial a_{jk} are determined according to the boundary conditions from the expressions:

$$a_{j0} = q_{j\text{s}};\, a_{j1} = a_{j2} = 0;\, a_{j3} = \frac{10(q_{j\text{e}} - q_{j\text{s}})}{T^3}$$

$$a_{j4} = \frac{15(q_{j\text{e}} - q_{j\text{s}})}{T^4};\, a_{j5} = \frac{6(q_{j\text{e}} - q_{j\text{s}})}{T^5}.$$

Taking into account the above coefficients, the law of change of the *j-th* generalized coordinate that ensures the movement of a grab to a given point in the workspace is written in the form:

$$q_j(t) = g_{j\text{s}} + \Delta g_j \tau^3 (10 + 15\tau + 6\tau^2) ,$$

where $\tau = t / T$ is the relative time of motion, is the displacement $\Delta q_j = q_{j\text{e}} - q_{j\text{s}}$ of the j-th generalized coordinate.

The duration of the time interval T is chosen from the condition that for each generalized coordinate the maximum values of velocities and accelerations do not exceed the maximum allowable values:

$$\left|\dot{q}_j\right|_{\substack{\max \\ j=1,n}} \leq \left|\dot{q}_j\right|_{\text{add}}; \left|\ddot{q}_j\right|_{\substack{\max \\ j=1,n}} = \left|\ddot{q}_j\right|_{\text{add}}.$$

To ensure this condition, the interval of travel time T must satisfy the condition:

$$T = \max_j \left[\max_{j=1,n}\left(1{,}878\frac{\left|q_{j\text{к}} - q_{j\text{s}}\right|}{\left|\dot{q}_j\right|_{\text{add}}}\right),\ \max_{j=1,n}\left(5{,}7735\frac{\sqrt{q_{j\text{к}} - q_{j\text{s}}}}{\left|\ddot{q}_j\right|_{\text{add}}}\right) \right]$$

This algorithm can be used to implement rectilinear motions of construction robots. The analysis of processing such movements by concrete layers showed the effectiveness of using approximating polynomials of order 5 to implement the command movements associated with the movement of the working body to a given point.

Variation laws of generalized coordinates q(t) based on polynomials of degree 3 are used for construction robots when the vectors of generalized coordinates at the initial q_s , final q_e and intermediate node points $q_k\,(k = 1, 2,...,\ m-1)$ are known.

This case is encountered when executing commands related to the movement of a working organ along a given trajectory, which is defined by a set of points. The number of intermediate points is determined by the nature of the trajectory and the required accuracy of its execution. The formation of laws of motion of generalized coordinates is based on the following requirements. The approximating polynomials must be able to pass through a sequence of vectors of generalized coordinates corresponding to each node point of the planned trajectory. In addition, for each generalized coordinate, continuity of the function itself and its first and second derivatives must be ensured. Fulfillment of these conditions ensures the continuity of velocities and accelerations and the smoothness of the trajectory to be realized [7].

Based on the initial information and requirements, a set of spline functions is constructed for each generalized coordinate:

$$P_j(t) = a_{j0} + a_{j1}t + a_{j2}t^2 + a_{j3}t^3.$$

The solution of this problem is performed on the basis of global interpolation. Two variants are possible: at zero velocities at the initial and final points of the motion trajectory; at given (nonzero) values of initial and final velocities of motion. The most interesting for construction robot control is the 1st method of spline construction. The input data in this case are: the trajectory of motion, defined by a set of node points; the values of velocity at the beginning and at the end of the trajectory; the value of the operating velocity along the trajectory. In addition, for each node point of the trajectory we set the orientation vector of the working body, as well as the constraints on the trajectory, on kinematics and dynamics of the manipulator. The limiting positions and admissible values of velocities and accelerations by degrees of mobility are considered as constraints. The purpose of the planning is to formulate laws of change $q_i(t);\dot{q}_i(t);\ddot{q}_i(t)$. The advantages of this planning method are its real-time execution and the direct generation of the controlled variables in the form of:

$$q_{ji}(t) = a_{ji0} + a_{ji1}(t - t_{i-1}) + a_{ji2}(t - t_{i-1})^2 + a_{ji3}(t - t_{i-1})^3, t_{i-1} \leq t \leq t_i,$$

where i = 1, 2,..., m is the number of the spline; j = 1, 2,...,n is the number of the degree of mobility of the generalized coordinate.

Determination of spline parameters is based on using the conditions of approximation of functions, continuity of splines and continuity of velocities and accelerations. For one degree of mobility, the system of equations that allows to determine the coefficients of splines has the form:

$$\begin{cases} P_i(t_i) = P_{i+1}(t_i) \rightarrow i = 1,2,...,m-1 \\ \dot{P}_i(t_i) = \dot{P}_{i+1}(t_i) \rightarrow i = 1,2,...,m-1 \\ \ddot{P}_i(t_i) = \ddot{P}_{i+1}(t_i) \rightarrow i = 1,2,...,m-1 \\ P_i(t_i) = q_i, P_1(t_0) = q_0 \end{cases}.$$

This system of equations is supplemented by initial conditions:

$$P_i(t) = \frac{M_{i-1}}{6\tau}(t_i - t)^3 + \frac{M_i}{6\tau}(t - t_{i-1})^3 + (q_{i-1} - M_{i-1}\frac{\tau^2}{6})\frac{t_i - t}{\tau} + (q_i - M_i\frac{\tau^2}{6})\frac{t - t_{i-1}}{\tau} \rightarrow t_{i-1} < t < t_i; i = 1, 2,..., m.$$

where M_{i-1}, M_i are the accelerations at the i-1 and i-th node points; the interpolation time at the i-th site.

The accelerations at the node points are determined on the basis of vector-matrix equations based on the condition of continuity of velocities at the node points and their equality to zero at the boundary points:

$$\begin{bmatrix} M_0 \\ M_1 \\ M_2 \\ ... \\ M_m \end{bmatrix} = \begin{bmatrix} 4 & \lambda_0 & 0 & 0 & . & . & . & 0 \\ \mu_1 & 4 & \lambda_1 & 0 & . & . & . & 0 \\ 0 & \mu_2 & 4 & \lambda_2 & . & . & . & 0 \\ . & . & . & . & . & . & . & . \\ 0 & . & . & . & . & . & \mu_m & 4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} d_0 \\ d_1 \\ d_2 \\ ... \\ d_m \end{bmatrix}$$

or $M = \Delta^{-1} \cdot D$.

The parameters of the right part of the equation are calculated using the formulas:

$$\lambda_0 = \mu_m = 2; \mu_i = \tau_i / (\tau_i + \tau_{i+1}); \lambda_i = 1 - \mu_i$$

$$d_0 = 3(q_1 - q_0)/\tau_1^2;\ d_m = 3(q_{m-1} - q_m)/\tau_m^2;$$

$$d_i = 3\left[(q_{i-1} - q_i)/\tau_{i+1} - (q_i - q_{i-1})/\tau_i\right]/(\tau_{i+1} + \tau_i).$$

When performing concrete laying as well as such technological operations as painting, priming, welding, cutting material, it is required not only to ensure the continuity and smoothness of movement, but also to plan the speed and acceleration of movement. In this case, when planning movements of the manipulator it is required to determine the speed of links

$$\dot{q}(t) = [\dot{q}_1(t), \dot{q}_2(t), ..., \dot{q}_n(t)]^T,$$

which provides the estimated technological speed of the tool during the development of the trajectory. It is necessary to formulate the laws of change of the generalized coordinates $q(t) = [q_1(t), q_2(t), ..., q_n(t)]$, if their values in the nodal points of the trajectory and the speeds of each degree of mobility in these points, corresponding to the technological speed of the tool movement are determined:

$$q_{ji}(t_i) = q_{ji};\ \dot{q}_{ji}(t_i) = \dot{q}_{ji} \rightarrow i = 1,2,..., n-1.$$

The law $q(t)$ is formed as a sequence of contiguous sections described by an interpolation polynomial. In this case, the cubic Hermite interpolation polynomial may be preferred. If at the node points t_i, except for function values $q_j(t_i) = q_{ji}$, we also know the values of derivatives $\dot{q}_j(t_i) = \dot{q}_{ji}$, then on each segment $[t_{i-1}, t_i]$ we can construct a spline according to the equation:

$$q_j(t) = q_{ji-1} \cdot \frac{(t_i - t)^2 \cdot (2(t - t_{i-1}) + \tau)}{\tau^3} + \dot{q}_{ji-1} \cdot \frac{(t_i - t)^2 \cdot (t - t_{i-1})}{\tau^2} + \\ + q_{ji} \cdot \frac{(t - t_{i-1})^2 \cdot (2(t_i - t) + \tau)}{\tau^3} + \dot{q}_{ji} \cdot \frac{(t - t_{i-1})^2 \cdot (t - t_i)}{\tau^2},$$

where $\tau = t_i - t_{i-1}$ is the interpolation step.

The feature of motion planning of manipulation systems in this case is that the time moments corresponding to the node points are initially defined, and the interpolation step is usually chosen uniformly. If the monotonicity of the functions is broken, the interpolation step should be changed and the calculation should be repeated.

4 Conclusion and results

The article presents for the first time a systematic approach to solving the problem of mathematical identification of robots for laying concrete mixture as objects of automatic control. For mathematical description there are multilink manipulators with a hydraulic drive, which most fully reflect the features and requirements of the performed operations, working environment conditions, transportation requirements and installation into a working condition. The feature of such manipulators is the presence of branched structures that provide the best kinematic and dynamic characteristics. The description of drive mechanisms in kinematic models is represented by a system of equations linking the parameters of the drive state with the generalized coordinates, velocities and accelerations. Analysis of methods for describing the dynamics of manipulating mechanisms has shown that for concrete-laying robots based on articulated manipulators, it is reasonable to build dynamic models by the method based on the Lagrange method formulated through the D'Alamber principle. This makes it possible to obtain the resulting equations in a convenient vector-matrix form and to realize an original approach to planning the trajectory of concrete-laying robots by the example of monolithic buildings and structures erection using sliding, repositionable and volumetric formwork. Since layers of concrete in this case are placed in a horizontal plane, the trajectory of a robotic concrete-laying head is reduced to the description of a sequence of movements in a plane parallel to the plane of the robot's coordinate system. In this case, all the requirements for preventing the appearance of singularity points in the system under consideration are observed, although this issue is not considered separately.

References

[1] Briot S., Arakelian V., Guégan S. Design and prototyping of a partially decoupled 4-DOF 3T1R parallel manipulator with high-load carrying capacity // Transaction of the ASME. Journal of Mechanical Design. 2008. V. 130. No. 12. P. 612−619.

[2] Arakelian V., Briot S., Glazunov V. Increase of singularity-free zones in the workspace of parallel manipulators using mechanisms of variable structure // Mechanism and Machine Theory. 2008. V. 43. P. 1129−1140.

[3] Hubert J., Merlet J.-P. Static of parallel manipulators and closeness to singularity // Transactions of the ASME. Journal of Mechanisms and Robotics. 2009. V. 1. P. 1−6.

[4] Bonev I., Zlatanov D., Gosselin C. Singularity analysis of 3-DOF planar parallel mechanisms via screw theory // Transactions of the ASME. Journal of Mechanical Design. 2003. V. 125. P. 573−581.

[5] Travush V., Erofeev V., Bulgakov A. and Buzalo, N. Mechatronic complex based on sliding formwork for the construction of monolithic high-rise buildings and tower-type structures made of reinforced concrete. IOP Conf. Series: Materials Science and Engineering 913 (2020) 022009 DOI:10.1088/1757-899X/913/2/022009

[6] Bulgakov A., Bock T., Otto J., Buzalo N. and Linner T. Requirements for Safe Operation and Facility Maintenance of Construction Robots. 2020 Proceedings of the 37th ISARC, Kitakyshu, Japan, pp. 369-376. DOI: doi.org/10.22260/ISARC2020/0053

[7] Lin Lee-Kuo. The research of automatic concrete placing. *In Proceedings of the 17th ISARC 2000,* pp. 1-5, Taipei, Taiwan.